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APPLICATION OF AN INVERSE DATA-DRIVEN MODEL FOR RECONSTRUCTING WHEEL MOVEMENT SIGNALS

Piotr Czop

AGH University of Science and Technology, Department of Robotics and Mechatronics, Al. Mickiewicza 30, 30-059 Kraków, Poland
(✉*piotr.czop@labmod.com, +48 502 026 501*)

Abstract

This paper considers a method for indirect measuring the vertical displacement of wheels resulting from the road profile, using an inverse parametric data-driven model. Wheel movement is required in variable damping suspension systems, which use an onboard electronic control system that improves ride quality and vehicle handling in typical maneuvres. This paper presents a feasibility study of such an approach which was performed in laboratory conditions. Experimental validation tests were conducted on a setup consisting of a servo-hydraulic test rig equipped with displacement, force and acceleration transducers and a data-acquisition system. The fidelity and adequacy of various parametric SISO model structures were evaluated in the time domain based on correlation coefficient, FPE and AIC criteria. The experimental test results showed that inverse models provide accuracy of inversion, ranging from more than 70% for the ARX model structure to over 90% for the OE model structure.

Keywords: Inverse model, parametric model, passenger vehicle, hydraulic damper.

1. Introduction

This paper considers a method for reconstructing wheel movement according to the road-load profile by means of an inverse, parametric data-driven model. The signal of the vertical wheel movement is required in semi-active suspension systems, which control the movement of the wheels via an onboard system, rather than the movement determined entirely by the surface on which the passenger vehicle is driving. The system, therefore, virtually eliminates body roll and pitch variation in many driving situations, including cornering, accelerating and braking [1]. This technology allows car manufacturers to achieve a higher degree of both ride quality and vehicle handling by keeping the tires perpendicular to the road when cornering, ensuring much higher levels of grip and control. An onboard computer detects wheel movement and body acceleration from sensors located throughout the vehicle. In addition, using data calculated by a "sky-hook" control technique, it controls the damping force of the hydraulic damper with the use of a mini servo-valve, which is used to bypass flow between internal chambers of the hydraulic damper.

The road profile generates a kinematic displacement excitation to the tire, which is transferred through the wheel assembly (see Fig. 1) to the lower arm, and further to the bottom mount (bracket) of the hydraulic damper. A hydraulic damper consists of a piston moving inside a liquid-filled cylinder to which the fixing bracket is attached. The top-mount is a rubber bearing element attached to the piston and the body of the vehicle. The excitation to the vehicle body which is transferred through the entire path consists of equivalent stiffness and damping sub-systems, corresponding to the tire, the wheel hub fixture, the hydraulic damper, and the top-mount (Fig. 1).
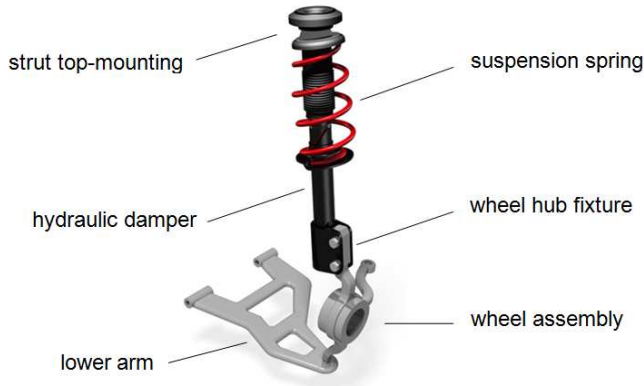
Fig. 1. A schematic view of a McPherson front strut suspension system.

A typical semi-active system uses displacement sensors which measure the relative distance between the rod and the hydraulic damper housing using electro-mechanical (LVDT) sensors. The velocity signal is obtained by differentiation of the displacement signal. The goal of this paper is to perform a feasibility study of indirectly reconstructing wheel vertical movement, based on the force measurement above the top-mount by means of dynamic model inversion. In turn, the displacement sensor is replaced by a load cell sensor, in which the force is converted into an electrical signal through deformation of a strain gauge. The strain changes the effective electrical resistance of the wire (e.g. four strain gauges in a Wheatstone bridge configuration). The electrical signal output is typically of the order of a few millivolts and requires calibration and amplification. This kind of sensor is a low-cost alternative to displacement sensors which are highly sensitive to dirt. The proposed approach was validated in the laboratory environment using road signals and a servo-hydraulic excitation system.

The structure of this paper is as follows. The second section presents the theory of model inversion and its underlying assumptions, as well as relevant aspects of system identification theory. The third section provides the results of experimental studies performed with the use of a servo-hydraulic test rig, while the last section discusses the results of model inversion, and summarizes the contributions of this work to inverse modeling of mechanical structures and systems.

2. Model inversion

This section provides a survey of the theory of model inversion and its applications towards predicting load by means of parametric models. The term "inverse" refers to the fact that the roles of the input and the output are exchanged, as opposed to the usual forward system structural dynamics problem. In this approach, the model is inverted to propagate the measured signal to obtain the input physical load. The graphical presentation of the inverse problem type can be found in Fig. 2.
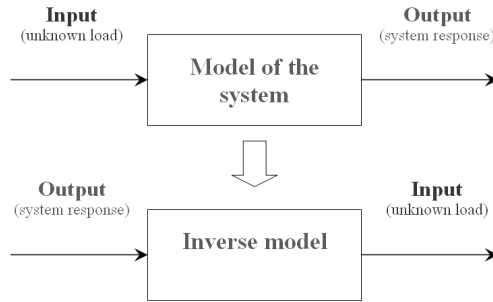
Fig. 2. Inverse problem presentation.

The methodology introduced here deals with a time-domain parametric approach to model inversion, the so-called dynamic correction/estimation, adopted mainly from control theory [2] and digital signal processing (DSP) theory [3].

The procedure of inverting a model does not correspond to any physical phenomenon and, therefore, inverse models always have a tendency to be unphysical. As all physical systems have a time delay as well as limited bandwidth, an exact inverse advances the signal as a result of the delay and amplifies high frequency noise without any bound, if the bandwidth is not restricted to the upper frequency band of the inversing load. This phenomenon is the so-called ill-posedness of the inversion related in this work to the inverse model stability and the bandwidth of the inverting load. Regularization techniques have to be used to incorporate efficient constraints into the inversion process and make it more well-posed to correctly estimate the load up to a certain bandwidth, typically given by the frequency at which amplitudes of the signal and the noise are equal. A standard solution of the regularization problem is to use (i) a stabilization method as discussed later in this Section and (ii) low-pass 'noise' filters. This fact suggests that the sampling rate of input and output signals has to be adapted to the maximal frequency of the reconstructed signal, although a very low sampling rate can result in severe aliasing. On the other hand, the sampling rate should allow the most important dynamics represented by vibration modes of the structure to be captured correctly. The significance of identified modes can be classified according to their energy levels while the structure is operating; which allows the most powerful ones to be selected and the sampling rate to be defined.

Reconstruction of the input of the system by inverting the system's model is important in multiple applications. Input reconstruction is a technique frequently used in the Internal Model Control (IMC) strategies [4] to invert data-driven parametric models and compensate the dynamics of the tracking process [5, 6], or for metrological purposes [7, 8]. The literature, however, rarely addresses the problem of dynamic model inversion [9] based on data-driven parametric models of mechanical structures and systems. Nonetheless, the technique (model inversion) is applicable to the problems of load reconstruction in mechanical systems in order to modify the dynamics of a structure or a system and to achieve better performance, e.g. to lower the level of loading forces [10-12]. Load prediction in systems for which the force signal cannot be directly measured due to constructional constraints [13], as in the case of forces being exchanged by a wheel and the road or rail, is considered to be one of the most practical applications of the inverse approach [14, 15]. One of the other applications, proposed by [16], is reconstruction of the moving force of a bridge using the method of adaptive input estimation.

A model and its inverse can have either a parametric or a non-parametric representation in time or in the frequency domain. A non-parametric representation uses a frequency response

function (FRF), called the spectral transfer function, or an impulse response function, while parametric representation uses a transfer function or state-space equations. Examples of applications of the frequency response function method are discussed in [17], while an impulse response function method is presented in [18, 10]. The parametric approach is considered as more adequate for real-time applications [19, 13]. For example, the inverse problem of load reconstruction of vehicles moving on a bridge requires time-varying parametric models to properly reconstruct the load [20, 16]. This approach is supported by recursive techniques well-known in system identification theory, e.g. the recursive least square (RLS) or the Kalman-filter approach. This paper considers a transfer function approach which is adequate for tracking displacement-load changes in the time domain, however the proposed model is time invariant. The advantage of such an approach is the immediate possibility of its application as a hardware pole-zero filter. The model structure has to be parameterized to reflect the dynamics of the system under consideration. If any a priori knowledge is not available, a blind search procedure for the best structure can be applied using key measures of the model quality, e.g. best fit, AIC.

A linear and time invariant system with a single input and a single output (SISO) can be represented as an input-to-output discrete transfer function

$$G(z) = \frac{y(z)}{u(z)} = \frac{b_0 + b_1 z + \ldots b_{nB} z^{nB}}{a_0 + a_1 z + \ldots a_{nA} z^{nA}} \ . \tag{1}$$

The role of the inverse model is to filter the response of the system in order to reconstruct the unknown input. It can be shown that the transfer function inverse to G is the reciprocal of G, i.e. the ratio

$$G^*(z) = \frac{1}{G(z)} = \frac{u(z)}{y^*(z)} = \frac{a_0 + a_1 z + \ldots a_{nA} z^{nA}}{b_0 + b_1 z + \ldots b_{nB} z^{nB}} \ . \tag{2}$$

Inversion requires all changes of the input signal u(z) of the direct model to be mapped on the output $y^*(z)$ of the inverse model. It can easily be seen that not all systems are invertible [6, 21, 3]. The transfer function is strictly proper if the degree of the numerator is less than the degree of the denominator $nB<nA$, while it is proper if the degree of the numerator is equal to the degree of the denominator, i.e. $nB=nA$. If the degree of the numerator is greater than the degree of the denominator, i.e. $nB>nA$, the transfer function is called improper [21, 6]. Inversion of a strictly proper transfer function requires compensation of the degrees of the numerator to be carried out with the use of so-called a reference transfer function $G_r(z)$.

$$G^*(z) = \frac{G_r(z)}{G(z)} = \frac{u(z)}{y^*(z)} = \frac{a_0 + a_1 z + \ldots a_{nA} z^{nA}}{b_0 + b_1 z + \ldots b_{nB} z^{nB}} \left(\frac{1-T_d}{z-T_d} \right)^{(nA-nB)}, \tag{3}$$

where T_d is the constant defining the quality of the inversion for signals of high-frequency content [21, 6, 11]. The inverse transfer function is unstable if at least one of the zeros of the direct transfer function is located outside the unit circle or inside the unit circle, for z^{-1} or z operators respectively. These zeros create a non-minimum phase transfer function, and hereafter are referred to as non-minimum phase zeros. The occurrence of non-minimum phase zeros is caused by one the following factors: (i) the sampling interval is too short, (ii) the discrete time delay is too long, or (iii) the number of poles, in comparison to the number of zeros, is too high [5]. It is clear from equations (2-3) that the inverse transfer function is

unstable because non-minimum phase zeros become unstable poles. The inverse transfer function can be stabilized, however, by factorization of the numerator B(z), as shown in [3, 12, 13].

$$B(z) = B_{stable}(z)B_{unstable}(z) ; \tag{4}$$

and substituting

$$\frac{B_{unstable}(1)}{B_{unstable}(z)} \text{ by } \frac{B_{unstable}(z^{-1})}{B_{unstable}(1)} . \tag{5}$$

Zeros of the polynomial $B_{stable}(z)$, called the stable polynomial, are exclusively the minimum phase zeros of the transfer function, while all the zeros of $B_{unstable}(z)$, called the unstable polynomial, are respectively the non-minimum phase zeros. The advantage of such a stabilization method is the lack of phase error and delay, while the only disadvantage is a small gain error that is, moreover, negligible if the output signal consists of low frequency components [3].

3. Experimental validation

The experiment discussed in this section aims at reconstructing a road load signal by means of inverting a direct model. The parameters of the direct model were identified based on another realization of that signal.

3.1. Measurement Conditions and Laboratory Hardware Configuration

Experimental tests were performed on a servo-hydraulic test-rig Hydropuls® MSP25 equipped with the IST8000 electronic controller. The test rig was used to load a hydraulic damper strut module (wheel hub fixture, hydraulic damper, spring and top-mount) and capture its dynamical characteristic, i.e. displacement vs. force. Data acquisition was performed with an 8-channel amplifier. The test rig is equipped with an oil supplying system (so called servo-pack) that provides a pressure of 28 MPa at a flow-rate of 90 l/min. The actuator provides 25 kN force at the rod, while the maximum stroke is 250 mm at the maximum achievable velocity of 2 m/s. The actuator rod is coupled to the adapter which transfers the force to a hydraulic damper mounted on a test rig. The main components of the servo-hydraulic system are the hydraulic actuator with the integrated displacement transducer in a piston-rod assembly (IST-Schenk) and a three-stage servo-valve system. The test rig is equipped with the PID-FF controller. The feed-forward (FF) section in this controller passes a proportion of the command signal to the controller output through a high-pass filter to block the command mean level. Different control settings are used depending on the type of the excitation signal. The excitation signal is converted into a voltage applied to the servo-valve which controls the amount of oil supplied to the chambers of the actuator. The road load data were used to simulate the ride conditions.

3.2. Inversion procedure

A block diagram of the procedure of inverting an SISO model is presented in Fig. 2. The initial step is the process of selecting an adequate model structure and an algorithm for estimating model parameters [11].
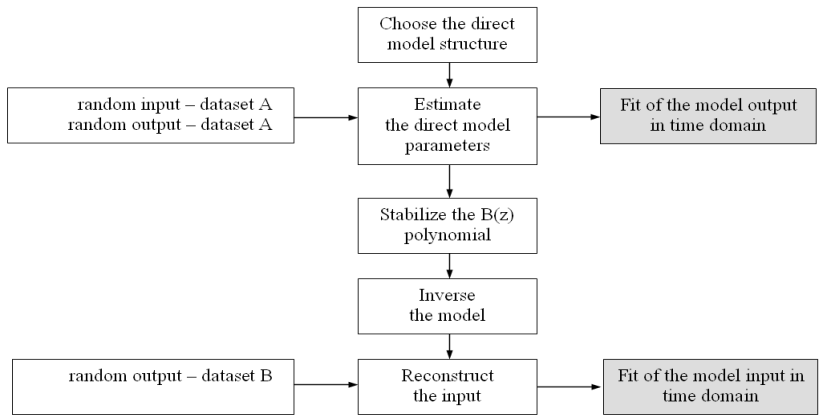
Fig. 3. Block diagram representation of the model inversion procedure.

The model structure is selected using the quality indicators of the direct and inverse models' fit to the data in the time domain. The selection process is supported by model quality measures and visual inspection as proposed in Table 1.

Table 1. Criteria of model structure optimality.

Criterion	Measure	Values
Fit in the time domain	Fit measure	Reconstructed input (output of the inverse model)
Fit in the time domain	Fit measure	Output of the direct model
Statistical properties of model residuals	FPE and AIC	Residuals of the direct model
Representation of modal properties	Subjective evaluation	Bode diagram and spectrum of residuals; both obtained from the direct model

The next step, estimation of the parameters of the selected model, is performed using the available input-output data, and a one-step-forward prediction of a direct model output is computed. If a priori information about the mechanical structure under consideration is unavailable or the structure is too complicated to be understood and correctly modeled using the first-principle approach, the adequacy of model structures is then evaluated by means of two measures, referred to in the literature as the Final Prediction Error (FPE) and the Akaike Information Criterion (AIC). The more accurate the model is, the smaller the values of the FPE and the AIC measures. Additionally, in order to detect the presence of abnormalities in the time/frequency domain, a visual inspection of the Bode plot of the input-to-output transfer path and the spectra of model residuals (the disturbance-to-output transfer path) was performed for each identified model structure. Analysis of the extensive quantity of such visual indicators (not presented here due to lack of space) indicates no presence of abnormalities. The major criterion for model order selection is, however, the comparison of the fit quality of the reconstructed inputs. The purpose of selection is to obtain a suitable inverse linear filter capable of providing the best possible reconstruction of the input signals with respect to the optimality criteria listed in Table 1.

Estimation of the parameters of the selected model, that is, estimation of the coefficients of the polynomials $A(z^{-1})$, $B(z^{-1})$, etc. from random data, was obtained from the Matlab System

Identification Toolbox [22]. A linear time-invariant (LTI) system, mapping a single input onto a single output (SISO) and in discrete time-steps, is represented by difference equations [23]. These equations take the form of (3), where $G(z^{-1})$ and $H(z^{-1})$ are discrete-time transfer functions containing adjustable coefficients and represent the input-to-output dynamics and the disturbance-to-output dynamics, respectively. The transfer functions $G(z^{-1})$ and $H(z^{-1})$ are rational functions of the operator z^{-1} that take the form shown on the right-hand-side of the equation [23]

$$y(i) = G(z^{-1})u(i) + H(z^{-1})e(i) = \frac{B(z^{-1})}{A(z^{-1})F(z^{-1})}u(i) + \frac{C(z^{-1})}{A(z^{-1})D(z^{-1})}e(i).$$ (6)

The polynomials $A(z^{-1})$, $B(z^{-1})$, $C(z^{-1})$, $D(z^{-1})$ and $F(z^{-1})$ are used for model parametrization. Special cases of the LTI SISO general model structure (3) are listed below as predefined model structures using the function notation to state their characteristic structural numbers [23];

$$
\begin{gathered}
PEM(nA, nB, nC, nD, nF, k) \\
BJ(nB, nF, nC, nD, k) \\
ARMAX(nA, nB, nC, k) \\
ARARX(dA, dB, dD, k) \\
OE(nB, nF, k) \\
ARX(nA, nB, k)
\end{gathered}
$$ (7)

where nA, nB, nC, nD and nF are polynomial orders and *k* is the input-to-output delay [23]. In case of an n-DOF system, for example the ARX(nA=2·n, nB=2·n) model structure can be used for the purpose of modeling the force-to-acceleration mapping, or the ARX(nA=2·n, nB=2·n-1) for the purpose of modeling the force-to-displacement or force-to-velocity mapping [24].

Stabilization of the polynomial $B(z^{-1})$ is an important stage of the inversion procedure and is achieved by reflecting the roots of $B(z^{-1})$ that have a magnitude greater than unity with respect to the boundary of the unit circle (i.e. to the inside of the unit circle) [3]. The *polystab(.)* function of the Matlab Signal Processing Toolbox is used for this purpose [22]. Upon stabilization, the model becomes invertible directly if the degree of the numerator of the input-to-output transfer function $G(z^{-1})$ is equal to the degree of its denominator, i.e. if the model is proper. If this is not the case, a reference transfer function [11] has to be used to compensate for the lower order of the numerator. The performance of the inversion algorithm can be evaluated either by visual inspection of the plot or by analyzing Pearson's product-moment correlation coefficient.

3.3. Identification and Inversion of the SISO Model

Inversion of the model was conducted according to the scenario shown in Fig. 3. The measured random displacement is required at the input of a direct model and the measured force is required at its output. The strategy implemented for optimizing selection of the model structure is the systematic search for a set of model structures that would satisfy the criteria listed in Table 1. For instance, a set of ARX(nA,nB,k) models were processed according to the procedure depicted in Fig. 3 in a manner ensuring that the criteria given in Table 1 were satisfied. The orders of polynomials were chosen so that $nA, nB \in \{5, 6, \ldots 16\}$, while the delay

k was fixed to the unity. The selected model structures are listed in Table 2 with results presented therein.

Table 2. Results of the input reconstruction and model fit to data.

Model	Direct model Fit %	Reconstructed displacement Fit %	FPE	AIC
BJ(13,7,7,15,1)	69%	88%	0.000556	-7.4917
OE(17,15,1)	78%	94%	0.053531	-2.9275
ARX(17,15,1)	69%	76%	0.000534	-7.5353
ARMAX(10,10,6,1)	68%	78%	0.000553	-7.5005
PEM(8,8,8,8,8,1)	67%	79%	0.000551	-7.5035
ARARX(15,13,6)	59%	72%	0.000547	-7.5673

To compare the results of direct measurements and those obtained with the procedure of model inversion, the correlation coefficients were computed for the random excitation. Besides Table 2, the exemplary results of reconstruction of displacement are presented graphically in Figs. 4-5 in the time domain. Before data processing, the signals were re-sampled down from 10 kHz to 1 kHz.
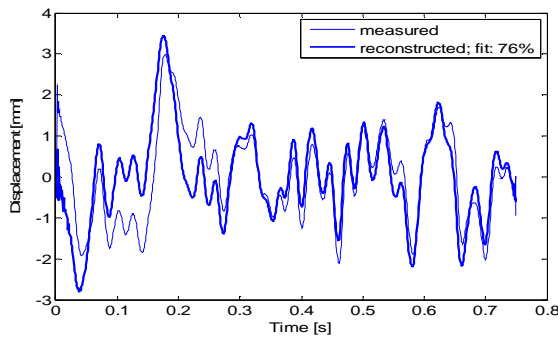

Fig. 4. Results of reconstruction of the displacement signal in the time domain with use of the ARX(17,15,1) model.
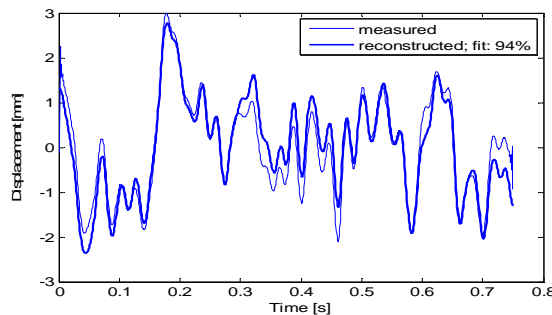

Fig. 5. Results of reconstruction of the displacement signal in the time domain with use of the OE(15, 17, 1) model.

4. Summary

The paper presents the case studies of inverting data-driven models of the suspension strut module using parametric SISO models. Experimental validation tests conducted on the servo-hydraulic test rig equipped with a data acquisition system, confirm that the methodology proposed herein, i.e. parametric system identification and model inversion, is valid. Results provided by data-driven parametric model structures are sufficient to constitute foundations for implementing them as inverse models in the form of fixed-point filters on a DSP platform. The model implemented in such a form is capable of filtering the responses of a mechanical system into a reconstructed input performing indirect measurements of the wheel displacement.

Nomenclature

i - discrete time
A, B, C, D, E, F - polynomials used for the representation of the transfer function
nA, nB, nC, nE, nF - order of polynomials used for the representation of the transfer function
z - operator of the Z transformation
e - disturbance variables in the system or its model
u - input variables in the system or its model
y - output variables in the system or its model
$G(z)$, $G(z^{-1})$ - discrete transfer function

Abbreviations

AIC – Akaike Information Criterion
ARMAX – AutoRegressive Moving Average with eXogeneous input
ARX – AutoRegressive with eXogeneous input
BJ – Box-Jenkins
DSP – Digital Signal Processing
FPE – Final Prediction Error
FRF – Frequency Response Function
LTI – Linear and Time-Invariant model/system
OE – Output Error
PEM – Prediction Error Method
SISO – Single-Input Single-Output
LVDT – Linear Variable Differential Transformer

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